Horizon boundary mechanics

Force, pressure, and work from the Einstein boundary potential

Horizon boundary mechanics resolves the Einstein boundary potential into a surface-stress response to area and a force–displacement response to acceleration length. Rindler and stationary Schwarzschild realize its canonical sector; Cai–Kim FRW supplies a cosmological screen realization.

1. Canonical boundary mechanics: force, pressure and work

For uniform stationary data on a prescribed-clock, nonexpanding Einstein null boundary ($\theta = \delta\theta = 0$), the selected scalar rate-source polarization gives [1]

$$\frac{\Theta_\kappa^{(0)}}{\Delta\tau} = \frac{Ac^2}{8\pi G}\,\delta\kappa = -F_L\,\delta L_\kappa, \qquad L_\kappa = \frac{c^2}{\kappa}, \qquad \kappa > 0. \tag{1}$$

Here L_κ is the light-travel length associated with timescale c/κ, not a universal proper distance. The established boost charge $J_\eta = Ac^3/(8\pi G)$ and normal rate $\Omega_\kappa = \kappa/c$ give [2]

$$\boxed{U_\partial = \Omega_\kappa J_\eta = \frac{Ac^2\kappa}{8\pi G}, \qquad U_\partial L_\kappa = cJ_\eta.} \tag{2}$$

Their differential is purely mechanical: $\Omega_\kappa\,dJ_\eta = \Pi_\partial\,dA$ and $J_\eta\,d\Omega_\kappa = -F_L\,dL_\kappa$, with $\Pi_\partial = U_\partial/A$. Hence

$$dU_\partial = \Pi_\partial\,dA - F_L\,dL_\kappa, \qquad F_L = -\left(\frac{\partial U_\partial}{\partial L_\kappa}\right)_A = \frac{U_\partial}{c^2}\kappa, \qquad \boxed{P_L \equiv \frac{F_L}{A} = \frac{\kappa^2}{8\pi G}}. \tag{3}$$

Area is intrinsic to the cut: a Rindler patch needs no radius or spherical projection. On a spherical marginal horizon, $E_{\rm MS} = c^4R/(2G)$ gives $U_\partial = E_{\rm MS}R/L_\kappa$; thus $U_\partial/E_{\rm MS} = R/L_\kappa$ is 1/2 for Schwarzschild and 1 for Cai–Kim FRW. Thermal inputs give $U_\partial = TS = F_LL_\kappa$; Schwarzschild Smarr reads $Mc^2 = TS + F_LL_\kappa$, each product $Mc^2/2$.

2. Cosmological application: the FRW apparent horizon

For spatially flat, expanding FRW, with $H = \dot a/a > 0$, select the instantaneous Cai–Kim calibration [3]:

$$R_A = \frac{c}{H} = L_\kappa, \qquad \kappa = cH, \qquad A = 4\pi R_A^2, \qquad V = \frac{4\pi R_A^3}{3}. \tag{4}$$

Use the standard Einstein–FRW Misner–Sharp identification $E_{\rm MS} = \rho V$ [4], where ρ is total bulk energy density. The boundary potential gives

$$E_{\rm MS} = \frac{c^4R_A}{2G} = \rho V = U_\partial = TS = F_LL_\kappa = 3P_LV. \tag{5}$$

Dividing by V relates the boundary pressure to the usual Friedmann constraint:

$$\boxed{\frac{c^2H^2}{8\pi G} = \frac{\kappa^2}{8\pi G} = P_L = \frac{\rho}{3}}, \qquad H^2 = \frac{8\pi G}{c^2}P_L = \frac{8\pi G}{3c^2}\rho. \tag{6}$$

Since $dV = A\,dR_A = A\,dL_\kappa$, the mechanical differential becomes

$$dE_{\rm MS} = T\,dS - P_L\,dV, \qquad F_L = \frac{E_{\rm MS}}{c^2}\kappa = \frac{c^4}{2G}. \tag{7}$$

This is a screen-state law, not a matter-flux law; P_L is distinct from the bulk fluid pressure.

Curvature, acceleration and conservation

With total bulk pressure p_m and Einstein coupling $\kappa_E = 8\pi G/c^4$,

$$\boxed{\frac{1}{R_A^2} = \kappa_EP_L}, \qquad \boxed{{}^{(2)}\mathcal{R}_\perp \equiv \frac{2\ddot a}{ac^2} = -\kappa_E(P_L + p_m)}. \tag{8}$$

Here $1/R_A^2$ is the sphere's Gaussian curvature and ${}^{(2)}\mathcal{R}_\perp$ is the normal time–radial Ricci scalar. Screen pressure sets the horizon-curvature scale; its sum with bulk pressure governs acceleration, which begins when $p_m < -P_L$. Bulk conservation becomes

$$\boxed{\dot P_L + H(\rho + p_m) = 0}, \qquad \rho = 3P_L. \tag{9}$$

The bulk enthalpy density $\rho + p_m$ controls the evolution of screen pressure.

Spatial curvature. The flat case is shown for clarity. For general FRW use $R_A = c/\sqrt{H^2 + kc^2/a^2}$ and $\kappa = c^2/R_A$. Then $P_L = \rho/3$ and Eqs. (5), (7)–(9) retain their form; $V = 4\pi R_A^3/3$ is areal volume, not generally proper spatial volume.

Inputs and scope. The null response fixes clock, cut, interval and other sources in the selected polarization. An evolving FRW screen is a separate calibrated realization, not an application of the nonexpanding-null theorem. Thermal identities use $T = \hbar\kappa/(2\pi ck_B)$ and $S = k_Bc^3A/(4G\hbar)$.

[1] Hopfmüller–Freidel, 1802.06135; Odak et al., 2309.03854. [2] Bianchi–Wieland, 1205.5325.

[3] Cai–Kim, hep-th/0501055. [4] Akbar–Cai, hep-th/0609128.

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