Horizon boundary mechanics

From two boundary coordinates to force and pressure

Start with the horizon boundary's area and acceleration length. The established Einstein boundary potential then yields surface stress, force, and pressure by one complete differential. The canonical calculation checks that the length response is present in the gravitational boundary action.

Notation in this reading edition: $L = L_\kappa$, $U = U_\partial$, $F = F_L$, $P = P_L$, and $\Pi = \Pi_\partial$. We use $\kappa > 0$ and a prescribed clock.

1. Two coordinates, one boundary potential

Area A measures transverse size. Acceleration length L expresses the selected normal surface-gravity rate κ as a length:

$$L = \frac{c^2}{\kappa}. \tag{1}$$

Like area, L is a shared mechanical coordinate across the realizations below—not a universal proper distance. Here c is the speed of light and G is Newton's constant. The boundary potential becomes

$$U = \frac{A\kappa c^2}{8\pi G} = \frac{Ac^4}{8\pi G L}. \tag{2}$$

Define the potential's mass equivalent, not an additional mass:

$$M_U \equiv \frac{U}{c^2} = \frac{A\kappa}{8\pi G} = \frac{Ac^2}{8\pi G L}. \tag{3}$$

2. Let the differential reveal the mechanics

Treat A and L as the two boundary source coordinates. Their complete differential gives

$$dU = \frac{U}{A}\,dA - \frac{U}{L}\,dL = \frac{\kappa c^2}{8\pi G}\,dA - M_U\kappa\,dL. \tag{4}$$

Read the coefficients as conjugate mechanical responses:

$$\Pi \equiv \left(\frac{\partial U}{\partial A}\right)_L = \frac{U}{A}, \qquad F \equiv -\left(\frac{\partial U}{\partial L}\right)_A = \frac{U}{L}. \tag{5}$$

$$\boxed{dU = \Pi\,dA - F\,dL}, \qquad F = M_U\kappa = PA. \tag{6}$$

Dividing the force by transverse area gives the normal-scale pressure:

$$\boxed{P \equiv \frac{F}{A} = \frac{\Pi}{L} = \frac{\kappa^2}{8\pi G} = \frac{c^4}{8\pi G L^2}}. \tag{7}$$

Π is force per length; P is force per area. No enclosed volume or temperature is needed for these definitions.

3. Check the same force in the canonical response

For uniform stationary data on a prescribed-clock, nonexpanding Einstein null boundary ($\theta = \delta\theta = 0$), the selected scalar rate-source response is [1]

$$\frac{\Theta_\kappa^{(0)}}{\Delta\tau} = \frac{Ac^2}{8\pi G}\,\delta\kappa = -\frac{A\kappa^2}{8\pi G}\,\delta L = -F\,\delta L. \tag{8}$$

This is the same length-work coefficient, now obtained from the gravitational action response. Rindler and stationary Schwarzschild realize this canonical sector. The FRW application below uses its separately selected Cai–Kim rate.

The same mechanics, three realizations

Each setting supplies its own rate and the relation between L and geometry. When the thermal identification applies, $U = TS$ and $\Pi\, dA = T\, dS$: temperature and entropy translate the mechanics rather than generate the force.

4. FRW: energy, pressure, and cosmic expansion

For spatially flat, expanding FRW, with $H = \dot{a}/a > 0$, choose the instantaneous Cai–Kim calibration [2]:

$$R_A = \frac{c}{H} = L, \qquad \kappa = cH. \tag{9}$$

With $A = 4\pi R_A^2$, $V = 4\pi R_A^3/3$, and the standard Einstein–FRW bulk-energy identification $E_{\rm MS} = \rho V$ [3], where ρ is total bulk energy density,

$$E_{\rm MS} = \rho V = \frac{c^4 R_A}{2G} = U = FL = 3PV. \tag{10}$$

$$\boxed{P = \frac{\rho}{3}}, \qquad H^2 = \frac{8\pi G}{c^2} P = \frac{8\pi G}{3c^2}\rho. \tag{11}$$

Because $dV = A\, dR_A = A\, dL$, the screen-state differential is

$$dE_{\rm MS} = \Pi\, dA - P\, dV = T\, dS - P\, dV. \tag{12}$$

Here $F = c^4/(2G)$. With bulk pressure p_m and Einstein coupling $\kappa_E = 8\pi G/c^4$, the horizon-curvature scale and cosmic acceleration are

$$\frac{1}{R_A^2} = \kappa_E P, \qquad \frac{2\ddot{a}}{ac^2} = -\kappa_E (P + p_m). \tag{13}$$

Bulk conservation becomes $\dot{P} + H(\rho + p_m) = 0$. Thus bulk enthalpy density $\rho + p_m$ controls the pressure's evolution.

5. Schwarzschild: the boundary potential is half the mass energy

With time normalized at infinity and $R_H = 2GM/c^2$, the same mechanics gives

$$L = 2R_H, \qquad U = TS = FL = \frac{Mc^2}{2}, \qquad M_U = \frac{M}{2}. \tag{14}$$

$$F = \frac{c^4}{8G}, \qquad P = \frac{c^4}{32\pi G R_H^2}, \qquad Mc^2 = TS + FL. \tag{15}$$

On a spherical marginal horizon, the energy ratio $U/E_{\rm MS} = R/L$ explains the contrast: FRW uses $L = R_A$, whereas Schwarzschild uses $L = 2R_H$.

6. Rindler: the construction needs no sphere

For a uniformly accelerated observer with proper acceleration $a_{\rm obs}$, select $\kappa = a_{\rm obs}$ and a finite planar horizon patch:

$$L = \frac{c^2}{a_{\rm obs}}, \qquad A = \Delta y\, \Delta z, \qquad P = \frac{a_{\rm obs}^2}{8\pi G}, \qquad U = \frac{A a_{\rm obs} c^2}{8\pi G}. \tag{16}$$

Here L is an actual proper horizon distance, while A and L remain independent. No enclosed source mass or spherical projection is needed.

Spatial curvature: the flat FRW case is shown for clarity. For general FRW use $R_A = c/\sqrt{H^2 + kc^2/a^2}$ and $\kappa = c^2/R_A$; $P = \rho/3$ remains unchanged. $V = 4\pi R_A^3/3$ is areal volume, not generally proper spatial volume.

[1] Hopfmüller–Freidel, arXiv:1802.06135; Odak–Rignon-Bret–Speziale, arXiv:2309.03854.

[2] Cai–Kim, arXiv:hep-th/0501055. [3] Akbar–Cai, arXiv:hep-th/0609128.

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